

Observed threshold anomalies as the first hope of a manifestation of Planck-length physics[†]

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ABSTRACT

The observations of photons from the BL Lac object Mk501 with energies above 10 TeV and of cosmic rays with energies above the GZK threshold appear to be inconsistent with conventional theories. Remarkably, among the recent new-physics proposals of solutions of these threshold paradoxes a prominent role has been played by proposals based on quantum properties of space-time. While the experimental evidence (and theory work attempting to interpret it) is much too preliminary to justify any serious hopes that we might have stumbled upon the first manifestation of a “quantum gravity”, the fact that for the first time phenomenological models involving quantum-gravity ideas are competing on level ground with other new-physics proposals clearly marks the beginning of a new stage of quantum-gravity research. I emphasize one important aspect of this new phenomenology: combining the determination of the relevant thresholds with data on the time/energy structure of gamma-ray bursts it is possible to distinguish between alternative quantum-gravity scenarios. This point is illustrated focusing on 3 specific scenarios: dispersion-inducing space-time foam, string-theory-motivated non-commutative space-time, and this author’s recent proposal of a relativistic theory in which the Planck length has the role of fundamental observer-independent minimum length.

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When Lee Smolin suggested that I contribute to the MG9 session he was to chair a lecture giving an “update” on the status of Planck-length phenomenology[1, 2], I was somewhat concerned. Lee clearly was hoping I could discuss significant new developments with respect to the status of Planck-length phenomenology described in my 1999 notes[1], but this phenomenology (which was thought to be impossible until very recently) remains very challenging and its progress should naturally have a very slow pace. My fear that there would not be any significant new developments for me to discuss proved to be unjustified when, just two months before MG9, Protheroe and Meyer[3] pointed out that some puzzling observations of photons from the BL Lac object Mk501 with energies above 10 TeV could be explained using the Planck-length-phenomenology model of Ref. [4]. This analysis by Protheroe and Meyer combines with a previous analysis by Kifune[5], which concerns another possibly related paradox in astrophysics data, in such a way that for the first time it appears legitimate to wonder whether some available (real-world!) data actually are a manifestation of Planck-length physics. Also significant for the outlook of Planck-length phenomenology is the fact that combining the study of these puzzling astrophysics data with the study of data on the time/energy structure of gamma-ray bursts it is possible to distinguish between alternative Planck-length-phenomenology models.

Let me start with a brief discussion of the experimental paradoxes analyzed in the mentioned studies by Protheroe and Meyer[3] and by Kifune.[5] The better known paradox is the one which Kifune (and, later, several other authors; see, *e.g.*, Refs. [6,7,8]) considered from the Planck-length-phenomenology viewpoint. This concerns the fact that Ultra High Energy Cosmic Rays (UHECRs) should interact with the Cosmic Microwave Background Radiation (CMBR) and produce pions. These interactions should forbid[9] the arrival on Earth of UHECRs with $E > 5 \times 10^{19}$ eV (the GZK limit), but several cosmic rays have been observed[10] with nominal energies at or above $10^{20} \pm 30\%$ eV. In Refs. [5-8] it was observed that this “threshold anomaly”[7] could be explained by assuming that the correct value of the threshold energies is not the one that follows from conventional Lorentz kinematics: values of the threshold that are consistent with the data can be obtained using deformed rules of kinematics based on the Planck-length-phenomenology proposal of Ref. [4].

The paradox which Protheroe and Meyer[3] (and, later, Refs. [7,11]) considered from the Planck-length-phenomenology viewpoint is somewhat analogous to the cosmic-ray paradox. HEGRA has detected[12] high-energy photons with a spectrum ranging up to 24 TeV from Mk501 (Markarian 501), a BL Lac object at a redshift of 0.034 (~ 157 Mpc). Also these observations are puzzling since a high energy photon propagating in the intergalactic space can interact with an IR background photon and produce an electron-positron pair if the product of the energies of the two photons exceeds m_e^2 . Pair production should forbid the arrival on Earth of Mk501 photons with energies above 10 TeV, in clear conflict with the mentioned observations of Mk501 photons with energies up to 24 TeV. (Using current IR background estimates Coppi and Aharonian[13] find an optical depth of 5 for 20TeV photons from Mk501.) In Refs. [3,7,11] it was observed that this Mk501 threshold anomaly could be explained by assuming that the correct value of the threshold energies is obtained using deformed rules of kinematics based on the Planck-length-phenomenology proposal of Ref. [4].

The UHECR and the Mk501 threshold paradoxes clearly have common features, although they involve different processes and different energy scales. There are various proposals that would provide new-physics interpretations of either the UHECR paradox[14, 15, 16, 17] or the Mk501 paradox[11], but the solution based on the Planck-length-phenomenology proposal of Ref. [4] receives some encouragement from the fact that, as Piran and I showed recently[7], there is a range of values of the

deformation scale (which is in principle a free parameter in the proposal of Ref. [4]) that solves simultaneously both paradoxes and this range of values is centered around the Planck length (justifying the interpretation as a quantum-gravity effect).

While it is still quite plausible that the correct solutions of the paradoxes might not even involve a deformation of the thresholds (for example, one of the proposed solutions of the UHECR paradox evaluates threshold energies in the conventional manner but advocates processes of decay of topological defects), in the remainder of this lecture I take as working assumption that the paradoxes do require a departure from the conventional way to evaluate threshold energies. Since I plan to discuss a few alternative scenarios for threshold shifts it is useful to quickly review the conventional evaluation of the threshold in the case of head-on collision between a soft photon of energy ϵ and momentum q and a high-energy particle of energy E_1 and momentum \vec{p}_1 leading to the production of two particles with energies E_2, E_3 and momenta \vec{p}_2, \vec{p}_3 . At threshold (no energy available for transverse momenta), energy conservation and momentum conservation imply

$$E_1 + \epsilon = E_2 + E_3 , \quad (1)$$

$$p_1 - q = p_2 + p_3 ; \quad (2)$$

moreover, using the ordinary Lorentz-invariant relation between energy and momentum (the *in vacuo* dispersion relation)

$$E^2 = p^2 + m^2 , \quad (3)$$

one also has the relations

$$\epsilon = q , \quad E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2p_i} , \quad (4)$$

where m_i denotes the mass of the particle with momentum p_i and the fact that p_1 (and, as a consequence, p_2 and p_3) is a large momentum has been used to approximate the square root.

The threshold value $p_{1,th}$ of the momentum p_1 can be identified with the requirement that the solutions for p_2 and p_3 as a function of p_1 (with a given value of ϵ) that follow from Eqs. (1), (2) and (4) should be imaginary for $p_1 < p_{1,th}$ and should be real for $p_1 \geq p_{1,th}$. This straightforwardly leads to the threshold equation

$$p_{1,th} \simeq \frac{(m_2 + m_3)^2 - m_1^2}{4\epsilon} . \quad (5)$$

As mentioned, the data on UHECRs and on Mk501 photons might suggest that this standard result underestimates $p_{1,th}$, at least for the processes $p + \gamma \rightarrow p + \pi$ and $\gamma + \gamma \rightarrow e^+ + e^-$. A result for the threshold different from (5) can of course be obtained by modifying the conditions for energy-momentum conservation (1)-(2) and/or the dispersion relation (3). Within Special Relativity it is of course only possible to introduce such modifications by assuming that propagation and scattering occur in presence of a background. It has long been conjectured that the Planck length might characterize one such background: the “quantum-gravity foam” or “quantum-gravity medium”. The presence of the background would allow to single out a preferred class of inertial frames, and it is therefore possible to assume, as commonly done[4, 18], that the dispersion relation is affected by a Planck-length deformation while energy-momentum conservation is unmodified.

I recently showed[19, 20] that another phenomenologically and conceptually consistent possibility is the one of a small Planck-length deformation of the Relativity postulates (basically fixing the Planck length as observer-independent “minimum length”[20]). This second scenario is of course more constrained: the fact that there is no background field associated with the Planck length (which implies that there is no preferred class of inertial observers for the description of the Planck-length structure of space-time) requires that the modifications of the dispersion relation and of energy-momentum conservation are correlated (both modifications must reflect the same transformation rules between different inertial frames).

The Planck-length phenomenology scenario which has been most carefully analyzed[5, 6, 7] with respect to the threshold anomalies is the one in which the Planck length is introduced together with an (unspecified) associated background, energy-momentum conservation is unmodified, and in the high-energy ($E \simeq p$) regime the dispersion relation is (in leading order in L_p) deformed according to

$$E^2 - \vec{p}^2 - m^2 \simeq \eta E^2 (L_p E)^\alpha \simeq \eta \vec{p}^2 (L_p E)^\alpha , \quad (6)$$

where E and \vec{p} are the energy and the (3-component) momentum of the particle, L_p is the Planck length ($L_p \sim 10^{-35}m$), and α and η are free parameters characterizing the deviation from ordinary Lorentz invariance. Experimental tests of the predictions of (6) were first proposed in Ref. [4], for the case $\alpha = 1$, and in Ref. [1], for generic α .

Kifune[5] considered the case $\alpha = 1$ and observed that for $\eta \simeq -1$ the new threshold was consistent with data on UHECRs. Aloisio *et al*[21] observed that the determination of the UHECR threshold and of the gamma thresholds of distant BL Lac objects could be used to test both the case $\alpha = 1$ and the case $\alpha = 2$. Protheroe and Meyer[3] considered the case $\alpha = 1$ and observed that for $\eta \simeq -1$ the new threshold was consistent with data on Mk501 photons. Piran and I[7] took as working assumption that both the UHECR and Mk501 paradoxes are indeed due to a threshold anomaly and tested the consistency of this assumption by studying the corresponding constraints on the α, η parameter space. We combined the (lower) bounds required by the working assumption for the threshold anomalies with the (upper) bounds on the same α, η parameter space which are imposed by the negative results of tests of (6) based on time-of-flight analyses[1, 4, 22, 23] of photons emitted by gamma-ray bursters and BL Lac objects. According to (6) one would predict energy-dependent relative delays between the times of arrival of simultaneously emitted photons; in fact, from (6) it follows that the speed of photons is energy-dependent: $v_\gamma = 1 + (1 + \alpha)\eta L_p^\alpha E^\alpha / 2$. We found that in order to solve both paradoxes and satisfy the time-of-flight upper bound it is basically necessary to have $\alpha \simeq 1$ and $\eta \simeq -1$. This strict constraint is mostly due to the Mk501 threshold anomaly, while the UHECR threshold anomaly is softer and would be consistent with all values of α in the range $1 \leq \alpha \leq 2$.

The fact that the UHECR threshold anomaly is softer than the Mk501 threshold anomaly is easily understood by looking at the formulas for the thresholds. Assuming (6) and assuming that energy-momentum conservation is unmodified one finds

$$p_{1,th} \simeq \frac{m_e^2}{\epsilon} + \eta \frac{p_{1,th}^{2+\alpha} L_p^\alpha}{4\epsilon} \left(\frac{1}{2^\alpha} - 1 \right) . \quad (7)$$

for $\gamma + \gamma \rightarrow e^+ + e^-$ (relevant for the Mk501 paradox), and

$$p_{1,th} \simeq \frac{(m_p + m_\pi)^2 - m_p^2}{4\epsilon} + \eta \frac{p_{1,th}^{2+\alpha} L_p^\alpha}{4\epsilon} \left(\frac{m_p^{1+\alpha} + m_\pi^{1+\alpha}}{(m_p + m_\pi)^{1+\alpha}} - 1 \right) . \quad (8)$$

for $p + \gamma \rightarrow p + \pi$ (relevant for the UHECR paradox). The pair-production threshold has simpler form because of the symmetry of the process, but in both cases the correction depends on the momentum scale of the process through $\eta p_{1,th}^{2+\alpha} L_p^\alpha / (4\epsilon)$. The coefficient of $\eta p_{1,th}^{2+\alpha} L_p^\alpha / (4\epsilon)$ is somewhat different in the two cases (it is smaller in the photopion production case) but the dominant difference comes from the fact that there are more than 5 orders of magnitude difference in $p_{1,th}$ ($p_{1,th}$ for UHECRs is more than 5 orders of magnitude greater than $p_{1,th}$ for Mk501), and therefore in comparing the magnitude of the corrections the dominant factor is the magnitude of $\eta p_{1,th}^{2+\alpha} L_p^\alpha / (4\epsilon)$. Since the (tentative) evidence of threshold anomalies suggests threshold shifts by factors of the same order (the Mk501 case requires a factor-2 upward shift, $10\text{TeV} \rightarrow 20\text{TeV}$, while the UHECR case requires a factor-6 upward shift, $5 \cdot 10^{19}\text{eV} \rightarrow 3 \cdot 10^{20}\text{eV}$) this leads to the conclusion that, for $|\eta| \simeq 1$, α cannot be much greater than 1 in order to explain the Mk501 paradox, while α as large as 2 could still explain the UHECR paradox.

The consistency of the overall picture relies also on the fact that we showed[7] that the region of the α, η parameter space that provides an explanation for both the UHECR and the Mk501 paradoxes is also consistent with the mentioned negative results of searches[1, 4, 22, 23] of time-of-flight anomalies, but the margin of consistency is relatively slim.[7] Sensitivity to time-of-flight anomalies is going to improve sharply in a few years with planned experiments such as the GLAST gamma-ray space telescope[24], and if these new searches still give negative results the α, η solution of the threshold paradoxes (at least the one for the Mk501 paradox) will be ruled out.

It is a significant development for Planck-length phenomenology that these UHECR and Mk501 paradoxes admit interpretation in terms of the scenario in which the Planck length is introduced together with an associated background, energy-momentum conservation is unmodified, and one uses the two-parameter Planck-length deformation of the dispersion relation (6). The experimental evidence (and theory work attempting to interpret it) is much too preliminary to justify any serious hopes that we might have stumbled upon the first manifestation of a “quantum gravity”, but the fact that for the first time phenomenological models involving quantum-gravity ideas are competing on level ground with other new-physics proposals clearly marks the beginning of a long-awaited new stage of quantum-gravity research. Moreover, even if the paradoxes are eventually understood in a way that does not involve anomalous thresholds, the result would still be significant since we would be able to rule out all pictures of the short-distance (Planckian) structure of space-time that predict anomalous thresholds. We finally do start having a few experiments that probe the short-distance structure of space-time with Planck-length sensitivity!

It is important to realize that the scenario in which the Planck length is introduced together with an associated background, energy-momentum conservation is unmodified, and one uses the two-parameter Planck-length deformation of the dispersion relation (6) is not the only way in which quantum properties of space-time may affect the evaluation of the thresholds. Whenever one considers the possibility of quantum space-time properties that introduce a new length scale (possibly the Planck length or the string length) together with an associated background it is natural to find that the dispersion relation is modified (reflecting the fact that the background allows to single out a preferred class of inertial frames). Besides the α, η example discussed in detail above, another scenario of this type which has attracted considerable attention recently is a string-theory scenario[25, 26] in which indeed new length scales (possibly, but not necessarily[25, 26], identified with the Planck or the string length) are introduced together with an accompanying background. A good effective-theory description of this string-theory scenario is obtained by describing all the new-physics

effects through space-time noncommutativity of the type $[X_\mu, X_\nu] = i\Theta_{\mu,\nu}$. The dimensionful (length-squared) parameters $\Theta_{\mu,\nu}$ are the only way in which the new background affects particle-physics processes. Preliminary results[26] appear to indicate that some particles acquire Θ -dependent corrections to the dispersion relation $E^2 = p^2 + m^2$ that go² like $(p^\mu \Theta_{\mu,\nu} p^\nu)^{-1}$. This $\Theta_{\mu,\nu}$ phenomenology would of course affect the determination of the thresholds in a way that is somewhat analogous³ to the α, η phenomenology discussed above. A detailed analysis of threshold anomalies in the $\Theta_{\mu,\nu}$ scenario is now in preparation.[27] Even before a detailed analysis one easily concludes that, since the α, η scenario predicts effects that increase with the particle momentum, while the $\Theta_{\mu,\nu}$ scenario predicts effects that decrease with the particle momentum, one of the two scenarios should emerge as experimentally favoured (or both will be ruled out), when eventually the UHECR and the Mk501 thresholds will be determined/understood and experiments on time-of-flight anomalies will be more accurate.

As I observed in recent work[19, 20], another logically consistent and phenomenologically viable possibility for the Planck length to characterize space-time structure is the one in which L_p has the role of observer-independent length scale. In the examples discussed above, the space-time-foam α, η scenario and the stringy $\Theta_{\mu,\nu}$ scenario, space-time length scales are introduced in a way that would allow to single out a preferred class of inertial observers, but it is also possible[19, 20] to attribute to L_p a role in Relativity which is completely analogous to the role of the velocity scale c : both c and L_p could be observer-independent and their role in space-time structure would not allow to distinguish between different inertial frames. In order to give an intuitive characterization of the implications of (and, particularly, the constraints imposed by) this drastic, but compellingly simple, assumption, let me analyze the dispersion relation (6), in the simple case of photons ($m = 0$) and $\eta = \alpha = 1$, $E^2 - \vec{p}^2 \simeq \vec{p}^2 L_p E$. If E and \vec{p} transform from one to another inertial frame according to ordinary Special Relativity, then this dispersion relation can only be valid in one preferred class of inertial frames, other inertial observers would not agree on the correction term (they would attribute to L_p a value that is different from the one obtained following Planck's prescription). It is however possible[19, 20] to add to the Relativity postulates the requirement that the dispersion relation $E^2 - \vec{p}^2 \simeq \vec{p}^2 L_p E$ is valid in all inertial frames, for fixed observer-independent value of L_p . The corresponding rules[19, 20] of transformation of E and \vec{p} are of course not exactly the ones of ordinary Special Relativity, but as long as the relative velocity between the inertial observers is not large the deformation is very mild (and in particular the new transformation rules reproduce the old ones in the small relative velocity limit).

If photons satisfy the dispersion relation $E^2 - \vec{p}^2 \simeq \vec{p}^2 L_p E$ in *all* inertial frames (with fixed observer-independent L_p), massive particles should accordingly satisfy in all inertial frames a dispersion relation of the type $E^2 - p^2 - m^2 \simeq F(E, p; m; L_p)$, with F some function such that (in leading order in L_p) $F(E, p; 0; L_p) = \vec{p}^2 L_p E$. For

²Of course, the IR singularity of $(p^\mu \Theta_{\mu,\nu} p^\nu)^{-1}$ reflects the fact that the effective theory breaks down in the IR limit. Still, it appears[26] that the effective theory might be reliable for momenta that are sufficiently soft for $(p^\mu \Theta_{\mu,\nu} p^\nu)^{-1}$ to have significant implications.

³Besides the different energy dependence of the effect, the $\Theta_{\mu,\nu}$ scenario differs from the α, η scenario also because it predicts a polarization dependence for the dispersion-relation deformation that applies to photons. A similar polarization dependence is also predicted by preliminary studies [18] of deformed dispersion relation in Loop Quantum Gravity. However, the effect described in Ref. [18] grows with energy (as in the α, η scenario) and can therefore be distinguished from the $\Theta_{\mu,\nu}$ scenario.

the simplest possibility, m -independent F (*i.e.* $F(E, p; m; L_p) = \bar{p}^2 L_p E$ for all m), I already verified the consistency of the overall scenario.[19] Work is now in progress[28] for other forms of F . The case of m -independent F is however sufficient to illustrate the implications of assuming that the deformed dispersion relation is valid in all inertial frames (rather than only in a specific class of inertial frames). In fact, the same dispersion relation $E^2 - p^2 - m^2 \simeq \bar{p}^2 L_p E$ which causes large threshold anomalies when considered in the sense of Ref. [4] (valid only in certain inertial frames), generates only very small threshold anomalies when it is assumed that it be valid in all inertial frames. The point is that if $E^2 - p^2 - m^2 \simeq \bar{p}^2 L_p E$ is valid in all inertial frames, then, as mentioned, the transformation rules between different inertial frames must be modified and the conditions for energy-momentum conservation must also be accordingly modified. The new conservation rules[19] are not very different from ordinary energy-momentum conservation (just like the dispersion relation has a Planck-length-suppressed deformation there is of course a corresponding Planck-length-suppressed deformation of the conservation rules) but they combine with the deformed dispersion relation in such a way that there is a nearly exact cancellation of deformation terms for the threshold, so that the threshold anomaly is much smaller than in the case in which ordinary energy-momentum conservation is assumed. Therefore experiments indicating explicitly a relatively large threshold anomaly would disfavour this scheme (but large threshold anomalies are not the only way to interpret the UHECR and Mk501 data: the observations could also be explained with sizeable anomalies just above threshold, where the mentioned cancellation is not efficient).

The implications of the deformed dispersion relation $E^2 - p^2 - m^2 \simeq \bar{p}^2 L_p E$ depend strongly on whether it is assumed to hold in all inertial frames (or just in one class of them) also for what concerns the mentioned time-of-flight anomalies. Searches[24] of time-of-flight anomalies should expect a dependence on the velocity of the source in the case in which the deformed dispersion relation singles out a preferred class of inertial frames (the velocity of the emitting galaxy with respect to the preferred frame would have absolute physical meaning in that scenario), while such a dependence should be absent if the Relativity postulates are consistently modified in such a way[19] that the deformed dispersion relation holds in all inertial frames.

Combining astrophysics data on possible threshold anomalies and the results of searches of time-of-flight anomalies we can therefore not only rule out several candidate short-distance (quantum) structures of space-time, but (if any anomalies are actually found/confirmed) we should be able to distinguish between different scenarios in which the Planck length is introduced together with an accompanying background (and a preferred class of inertial frames), *e.g.* we should be able to distinguish between the α, η scenario and the $\Theta_{\mu, \nu}$ scenario with preferred class of inertial frames, and we should also be able to distinguish between the possibility that the Planck scale is introduced together with an accompanying preferred class of inertial frames and the possibility that the Planck length is introduced as an observer-independent characteristic of space-time structure, *e.g.* we should be able to distinguish between the case in which the α, η dispersion relation is valid only in a preferred class of inertial frames and the case in which the α, η dispersion relation is valid in all inertial frames.

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